Hey, 2019-2020 CP Calculus friends!

Are you ready for August?!?! Kidding. I know you’re excited to take a break and, to be quite honest, I’m looking forward to some rest as well!

FYI! If you haven’t heard, the cool thing about Calculus is that it’s a course in which all your previous math knowledge can come back into play at any given moment! It’s often a misuse of algebraic and/or trig ideas that causes students to make errors rather than a misunderstanding of new Calculus content. In hopes of keeping you ‘fresh’ for the upcoming school year, I’ve compiled some problems for you to work through at your leisure during the summer.

So, ‘what are the expectations for these problems,’ you might ask? Well, you’re welcome to collaborate with classmates and seek online support (www.khanacademy.org is an excellent one, of course), but I would highly encourage you to work independently on the items FIRST. Remember that success is as valid as struggle AND will give us all insight into how to best use the first few weeks of the school year! So, above all else, approach the problems with integrity and make note of places where you felt most challenged.

Feel free to email me if you have questions or concerns. (I will try to check school email on a weekly basis over the summer.)

Happy Problem Solving! 😊

Mrs. Prowell

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CP Calc Summer Assignment

Please do your work for each problem on notebook paper.

1. Let \( f(x) = \sqrt{x + 1} + 4 \).
   a. The domain of \( f \) is
   b. \( f(3) = \)
   c. \( f(t^2 - 1) = \)
   d. \( f(x) = 7 \) if \( x \) is
   e. The range of \( f \) is

Find the domain of each function. Express your answer in interval notation.

2. \( f(x) = \frac{x^2 - 1}{x+1} \)

3. \( h(x) = \sqrt{x - 3x^2} \)

4. \( h(x) = \frac{3}{2 - \cos x} \)

5. \( f(x) = x^3 + 2 \)

6. \( h(x) = 3 \sin x \)

7. \( h(x) = 3 + \sqrt{x} \)

8. Use the equation \( y = x^2 - 6x + 8 \) to answer the following questions.
   a. For what values of \( x \) is \( y = 0 \)?
   b. For what values of \( x \) is \( y = -10 \)?
   c. For what values of \( x \) is \( y \geq 0 \)?
   d. Does \( y \) have a minimum value? If so, find it.
   e. Does \( y \) have a maximum value? If so, find it.

9. Express the length \( L \) of a chord of a circle with radius 10 cm as a function of the central angle \( \theta \).
   (Hint: Think Law of Cosines.)
10. An open box is to be constructed from a rectangular sheet of metal 8 inches by 15 inches, by cutting out squares of length $x$ from each corner and bending up the sides.
   a. Express the volume $V$ as a function of $x$.
   b. Graph the function $V$ and estimate its range.
   c. Discuss how you might use all of this information to construct a box of maximum volume.

11. The graph of $y = 1 + (x - 2)^2$ may be obtained by shifting the graph of $y = x^2$ to the ________ (left/right) by _______ unit(s) and then shifting this new graph ________ (up/down) by _______ unit(s).

12. Let $f(x) = \begin{cases} |x + 1|, & -2 \leq x \leq 0 \\ |x - 1|, & 0 < x \leq 2 \end{cases}$
   a. Which letter of the alphabet most resembles this piecewise function?
   b. Is $f$ an even function? Why or why not?

13. The graph of a function $f$ is shown in the accompanying figure. Sketch the graphs of the following equations.
   a. $y = f(x) - 1$
   b. $y = f(x - 1)$
   c. $y = \frac{1}{2} f(x)$
   d. $y = f(-\frac{1}{2}x)$

Sketch the graph of the equation by translating, reflecting, compressing and stretching the graph of $y = x^2$ appropriately, and then use a graphing utility to confirm that your sketch is correct.
14. $y = -2(x + 1)^2 - 3$
15. $y = x^2 + 6x$

Sketch the graph of the equation by translating, reflecting, compressing and stretching the graph of $y = \sqrt{x}$ appropriately, and then use a graphing utility to confirm that your sketch is correct.
16. $y = 3 - \sqrt{x + 1}$
17. $y = \frac{1}{2} \sqrt{x + 1}$
Sketch the graph of the equation by translating, reflecting, compressing and stretching the graph of 
\( y = \frac{1}{x} \) appropriately, and then use a graphing utility to confirm that your sketch is correct.

18. \( y = \frac{1}{x - 3} \)
19. \( y = 2 - \frac{1}{x + 1} \)

Sketch the graph of the equation by translating, reflecting, compressing and stretching the graph of 
\( y = |x| \) appropriately, and then use a graphing utility to confirm that your sketch is correct.

20. \( y = |x + 2| - 2 \)
21. \( y = 1 - |x - 3| \)

22. \( f(x) = 2\sqrt{x - 1}, g(x) = \sqrt{x - 1} \). Find formulas for \( f + g, f - g, fg \) and \( \frac{f}{g} \) and state the domains of the functions.

23. Let \( f(x) = x^2 + 1 \). Find:
   a. \( f(t^2) \)
   b. \( f(x + 2) \)
   c. \( f(x + h) \)
   d. \( f(-x) \)
   e. \( f(\sqrt{x}) \)
   f. \( f(3x) \)

Find formulas for \( f(g) \& g(f) \) and state the domains of the compositions.

24. \( f(x) = x^2, g(x) = \sqrt{1 - x} \)
25. \( f(x) = \frac{1 + x}{1 - x}, g(x) = \frac{x}{1 - x} \)

26. Find a formula for \( f \left( g \left( h(x) \right) \right) \) given \( f(x) = x^2 + 1, g(x) = \frac{1}{x}, h(x) = x^3 \).

Decompose \( f \) into two functions \( g \) and \( h \) such that \( f(x) = g \left( h(x) \right) \).

27. \( f(x) = \sqrt{x + 2} \)
28. \( f(x) = \sin^2 x \)
Find \( \frac{f(x+h)-f(x)}{h} \). Simplify each ratio as much as possible.

29. \( f(x) = 3x^2 - 5 \)
30. \( f(x) = \frac{1}{x} \)

Classify each function as even, odd or neither.

31. \( f(x) = x^2 \)
32. \( f(x) = x^3 \)
33. \( f(x) = |x| \)
34. \( f(x) = 2 \)

Match the equation with the correct graph. Then find the equations of the horizontal and vertical asymptotes.

35. \( y = \frac{x-1}{x^2-x-6} \)

36. \( y = \frac{x^2}{x^2-x-2} \)

Find an equation of the form \( y = D + A \sin Bx \) or \( y = D + A \cos Bx \) for each graph.

37.

38.
39. A student enters a number on a calculator, doubles it, adds 8 to the result, divides the sum by 2, subtracts 3 from the quotient and then cubes the difference. If the resulting number is \( x \), what was the student's original number?

40. If \((3, -2)\) is a point of the graph of an odd invertible function \( f \), then _______ and _______ are points of the graph of \( f^{-1} \).

Determine the exact value(s) of each trig function without using a calculator.

41. \( \sin^{-1}(-1) \)
42. \( \tan^{-1}(1) \)
43. \( \cos^{-1}\left(\frac{1}{2}\right) \)

Determine whether \( f \) and \( g \) are inverse functions.

44. \( f(x) = 4x, \ g(x) = \frac{1}{4}x \)
45. \( f(x) = 3x + 1, \ g(x) = 3x - 1 \)
46. \( f(x) = \sqrt[3]{x - 2}, \ g(x) = x^3 + 2 \)
47. \( f(x) = x^4, \ g(x) = \sqrt[4]{x} \)

Find \( f^{-1}(x) \).

48. \( f(x) = 7x - 6 \)
49. \( f(x) = 3x^3 - 5 \)

50. Given that \( \theta = \tan^{-1}\left(\frac{4}{3}\right) \), find the exact values of \( \sin \theta \), \( \cos \theta \), \( \csc \theta \), \( \sec \theta \) & \( \cot \theta \).

51. Given a triangle with sides \( a = 2, b = 3, c = 4 \), find the measure of angle \( C \) to the nearest tenth of a degree. (Hint: Think Law of Cosines!)

Express each as a power of 4.

52. 1
53. 2
54. \( \frac{1}{16} \)
55. \( \sqrt{8} \)
56. 5
Solve for $x$.

57. $7e^{3x} = 56$
58. $\ln x = 3$
59. $\log(x - 1) = 2$
60. $2\log x - \log(x + 1) = \log 4 - \log 3$
61. $\log_5(5^{2x}) = 8$
62. $5^{-2x} = 3$
63. $e^{2x} - e^x = 6$

Find the exact value of the expression without using a calculator.

64. $\log_2 16$
65. $\log_2 \frac{1}{32}$
66. $\log_4 4$
67. $\log_9 3$

Use your knowledge of log properties to rewrite the expression in terms of $r$, $s$ and $t$ where $r = \ln a$, $s = \ln b$ and $t = \ln c$.

68. $\ln a^{2\sqrt{bc}}$
69. $\ln \frac{b}{a^2c}$

70. Condense $4\log 2 - \log 3 + \log 16$ into a single log expression.

71. The equation $Q = 12e^{-0.055t}$ gives the mass $Q$ in grams of radioactive potassium-42 that will remain from some initial quantity after $t$ hours of radioactive decay.

   a. How many grams were there initially?
   b. How many grams remain after 4 hours?
   c. How long will it take to reduce the amount of radioactive potassium-42 to half of the initial amount?